2.2

5) $\lim_{y \to 2^-} \frac{(y-1)(y-2)}{y+1} = 0$

$\frac{4-6+2}{3}$

$\frac{-2+2}{3}$

0

$\frac{y^2-3y+2}{y+1}$

$\frac{(2-1)(2-2)}{(2+1)}$

$\frac{(2)^2-3(2)+2}{(2)+1}$

$= 0$

substituting?
isn't that
Cheating?

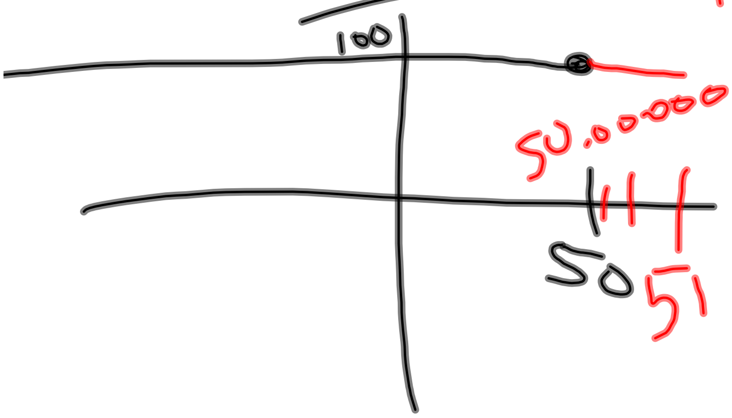
a) the only thing I
know how to do

b) polynomials
exponential
rational
logarithmic
trigonometric

$\Rightarrow$ substitution WORKS
as long as x is
in domain

$$f(x) = \begin{cases} 100 & ; x \leq 50 \\ \text{und} & ; x > 50 \end{cases}$$

1 pt "tail" on the right



$$\lim_{x \to 50^-} f(x) = 100$$

$$\lim_{x \to 50^+} f(x) = \text{DNE}$$

$$\lim_{x \to 4} \frac{\cancel{x}^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}} = \lim_{x \to 4} x + 4$$

$$4 + 4 = 8$$

$\frac{0}{0}$ MEANS YOU DON'T KNOW ANYTHING

$$\lim_{x \to 1^+} \frac{x^4 - 1}{x - 1} \qquad \frac{(x^2+1)(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$(1^2+1)(1+1)$$

$$2(2) = 4$$

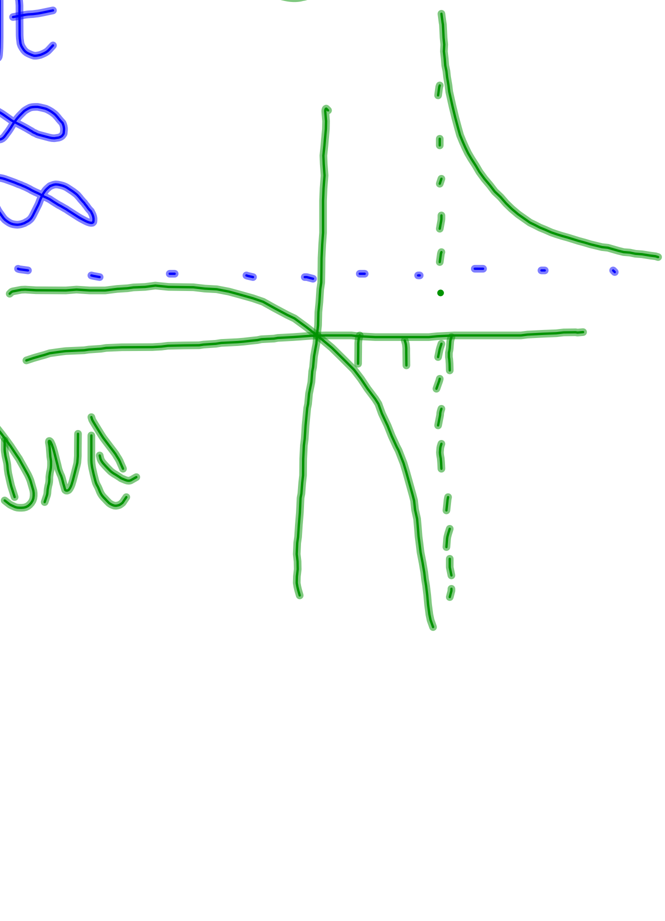
15) $\lim_{x \to 3^+} \frac{x}{x-3}$ $\quad \frac{3}{0} + \infty$

$\frac{x}{x} = 1 \quad = \text{DNE}$

$+\infty$

$-\infty$

$\lim_{x \to 3^-} f(x) = -\infty$



$\lim_{x \to 3} f(x) = \text{DNE}$

13)

$$\lim_{t \to 2} \frac{t^3+3t^2-12t+4}{t^3-4t}$$

$$\frac{\overset{8}{2^3}+\overset{12}{3(2)^2}-\overset{24}{12(2)}+\overset{4}{4}}{\underset{8}{2^3}-\underset{8}{4(2)}}$$

$f(2)=0$

$(x-2)P(x)=f(x)$

$$\frac{0}{0}$$

$$\lim_{t\to 2} \frac{t^3+3t^2-12t+4}{t^3-4t}$$

$$\begin{array}{r|l} & t^2+5t-2 \\ \hline t-2 & t^3+3t^2-12t+4 \\ & -(t^3-2t^2) \\ \hline & \quad 5t^2-12t+4 \\ & -(5t^2-10t) \\ \hline & \qquad -2t+4 \end{array}$$

$\cancel{t-2 \,\big|\, t^3-4t}$

t^3-4t

$=t(t^2-4)$

$$\lim_{t\to 2} \frac{t^2+5t-2}{t(t+2)} = \frac{4+10-2}{2(4)} = \frac{12}{8}$$

$=\frac{3}{2}$

27) $\lim\limits_{x \to 2^+} \dfrac{1}{|2-x|}$ DNE, $+\infty$, $\cancel{-\infty}$

$$= +\infty$$

$$\frac{1}{|2-x|} = \begin{cases} \dfrac{1}{2-x}, & 2-x \geq 0 \\ & \quad +x \quad +x \\ & \quad 2 \geq x \\ \dfrac{1}{-(2-x)}, & 2-x < 0 \end{cases}$$

$$= \begin{cases} \dfrac{1}{2-x}, & x \leq 2 \\ \dfrac{1}{-(2-x)}, & x > 2 \end{cases}$$

BUT $x \neq 2$

$$f(x) = \begin{cases} \dfrac{1}{2-x}, & x < 2 \\ \dfrac{1}{x-2}, & x > 2 \end{cases}$$

$$Y_1 = (1/(2-x))(x<2) +$$
$$(1/(x-2))(x>2)$$

5)

$$\lim_{y \to 2^-} \frac{(y-1)(y-2)}{y+1} = 0$$

1.999

$$\boxed{\frac{(2-1)(2-2)}{2+1}} = \frac{(1)(0)}{3} = 0$$

Substituting?
Isn't that cheating?

Substituting WORKS
for

- polynomials
- rational functions
- exponential $f^{\underline{n}}$
- logarithmic $f^{\underline{n}}$
- trigonometric $f^{\underline{n}}$

AS LONG AS the value is in the DOMAIN

$$\lim_{x \to 4} \frac{x^2-16}{x-4} = \lim_{x \to 4} \frac{(\cancel{x-4})(x+4)}{\cancel{x-4}}$$

$$\lim_{x \to 4} x+4 = 8$$

$$\frac{0}{0} \equiv \text{I don't know anything}$$

⑨

$$\lim_{x \to 1^+} \frac{x^4-1}{x-1} = \frac{(x^2-1)(x^2+1)}{x-1} = \frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{x-1}}$$

$$(x+1)(x^2+1)\Big|_{x=1} = 2 \cdot 2 = 4$$

$$\lim_{x \to 1^+} \frac{x^4-1}{x-1} = 4$$

$$f(2)=0$$

$$f(x)=(x-2)P(x)$$

$$\lim_{t\to 2}\frac{(t^3+3t^2)-(12t+4)}{t^3-4t}$$

$$P(x)=(x-a)(x-b)(x-c)$$

$$P(2)=(2-a)(2-b)(2-c)=0$$

$$
\begin{array}{r}
t^2+5t-2 \\
t-2\,\overline{)\,t^3+3t^2-12t+4} \\
-(t^3-2t^2) \\
5t^2-12t \\
-(5t^2-10t) \\
-2t+4 \\
-(-2t+4)
\end{array}
$$

$$\lim_{t\to 2}\frac{t^3+3t^2-12t+4}{t^3-4t}=\lim_{t\to 2}\frac{\cancel{(t-2)}(t^2+5t-2)}{t\cancel{(t-2)}(t+2)}$$

$$=\frac{12}{8}=\frac{3}{2}$$

SIGN SINE

$$\lim_{x\to 3^+} \frac{x}{x-3} \qquad \lim_{x\to 3^+} \frac{3}{3-3} = \frac{3}{0}$$

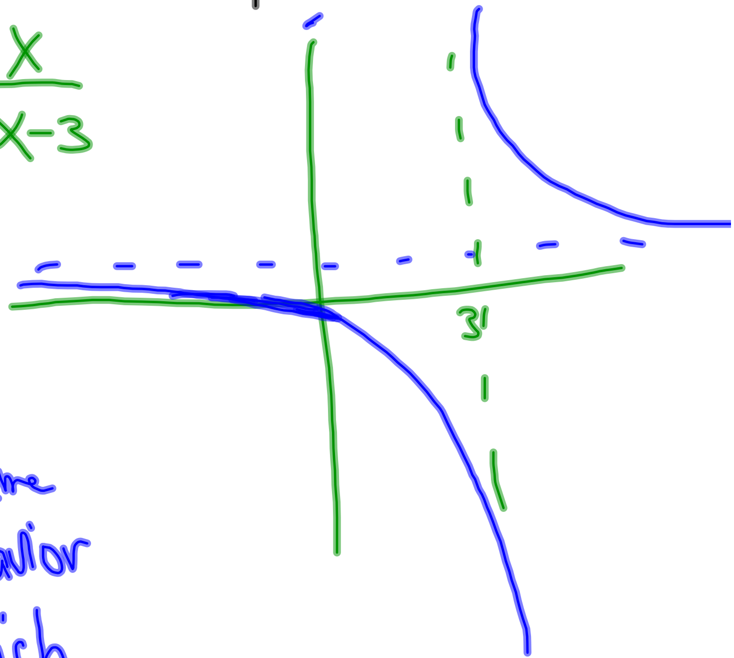
$+\infty$

DNE

$+\infty$

$-\infty$

$$\frac{x}{x-3}$$



$x^7 - 100000000\,x^6 + \ldots$

$$\frac{x}{x-3}$$

$$\frac{x}{x} = 1$$

Polynomials have the same end-behavior as the high degree term

$$\lim_{x\to 2^-} \frac{x}{x^2-4} = -\infty$$

19 $\quad \dfrac{x}{x^2-4}$

VA: $x^2-4=0$

$(x-2)(x+2)=0$

$x=-2,+2$

HA:

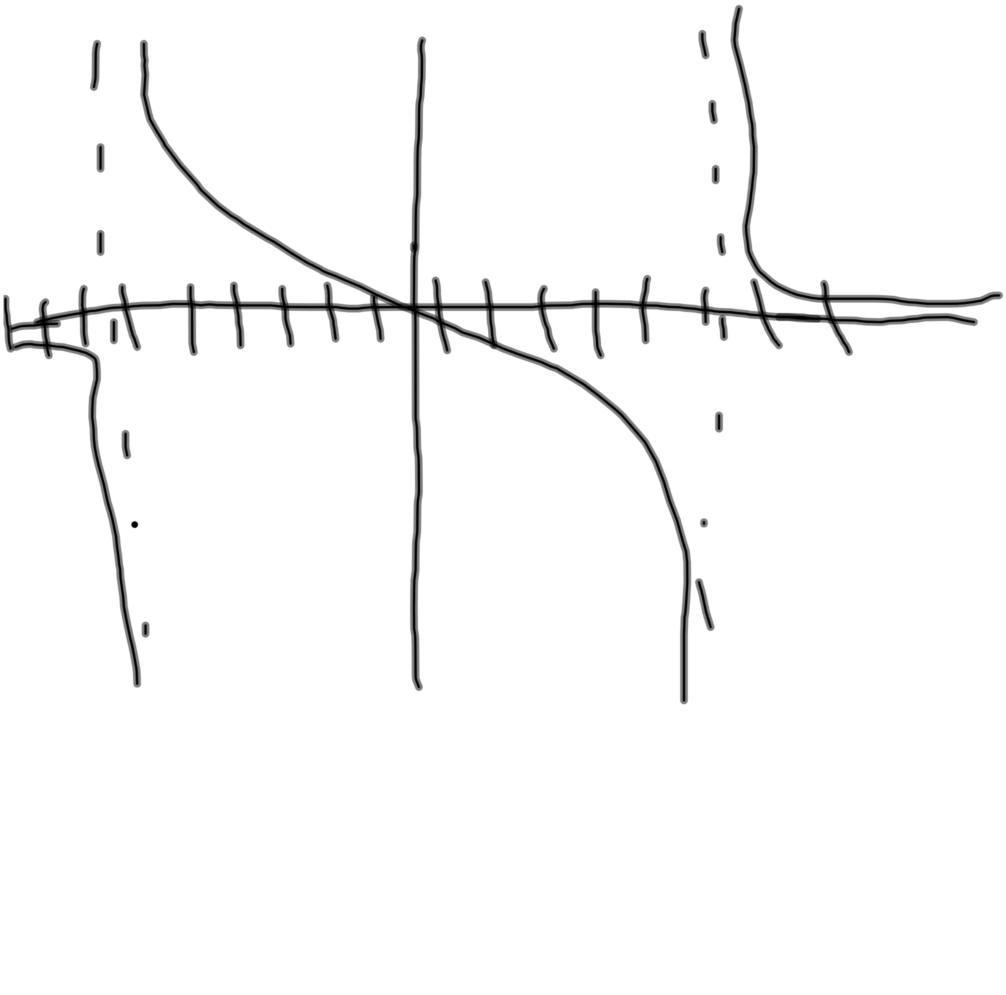
$\approx$ e.b. of $\dfrac{x}{x^2} = \dfrac{1}{x}$

$y=0$

21) $\lim_{y \to 6^+} \frac{y+6}{y^2-36}$ $\lim_{y \to 6^+} \frac{\cancel{y+6}\,1}{\cancel{y+6}\; y-6}$ $\frac{y}{y^2}$ $\frac{1}{y}$

VA: 6, -6

HA:



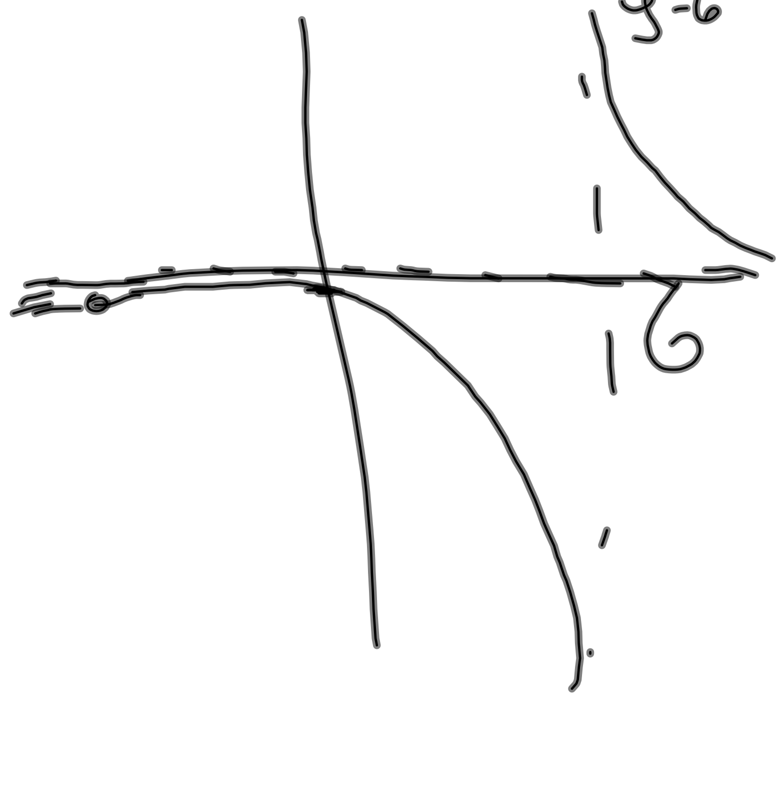
$$\frac{y+6}{y^2-36} = \frac{y+6}{(y+6)(y-6)}$$

$$\frac{\cancel{x+6}}{(x+6)(x-6)}$$

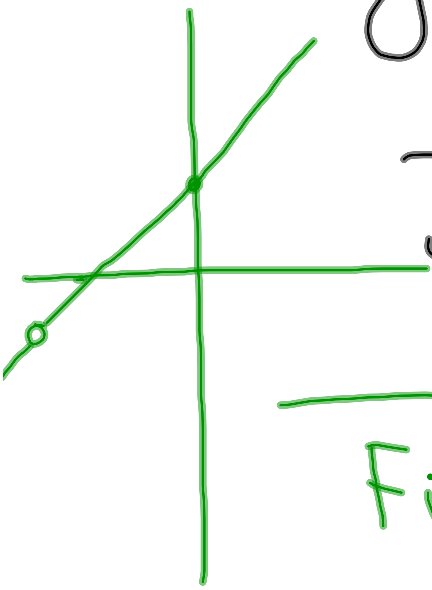
$$\frac{1}{y-6}$$

VA: $y=6$

HA: $x: \frac{y}{y^2} = \frac{1}{y} \to 0$



Draw a graph of a function of $y = x+2$



BUT make it have a hole at $x = -3$

Find an expression or rule that does same thing...

$$f(x) = \frac{(x+2)(x+3)}{(x+3)}$$

29

$$\lim_{x \to 9} \frac{\overset{(\sqrt{x}-3)(\sqrt{x}+3)}{x-9}}{\sqrt{x}-3} \cdot \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)}$$

CONJUGATE

$$\frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{x-9}}$$

$$\lim_{x \to 9} \sqrt{x}+3 = 6$$

$$3+3$$

27) $$\lim_{x \to 2^+} \frac{1}{|2-x|}$$

25) $f(x) = \dfrac{3-x}{x^2-2x-8}$

?

HA

end behavior like

$\dfrac{-x}{x^2} = \dfrac{-1}{x}$

$y=0$

VA:

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=-2,+4$

$f(x)=0$

$3-x=0$

$x=3$

y-int

$f(0)=\dfrac{3}{-8}$

TEST

$\dfrac{3-x}{(x-4)(x+2)} > 0$

$(3,4)$ open

$3<x<4$

Identify "cut points"

num=0 $\qquad$ den=0

$x=3$ $\qquad$ $x=-2,4$

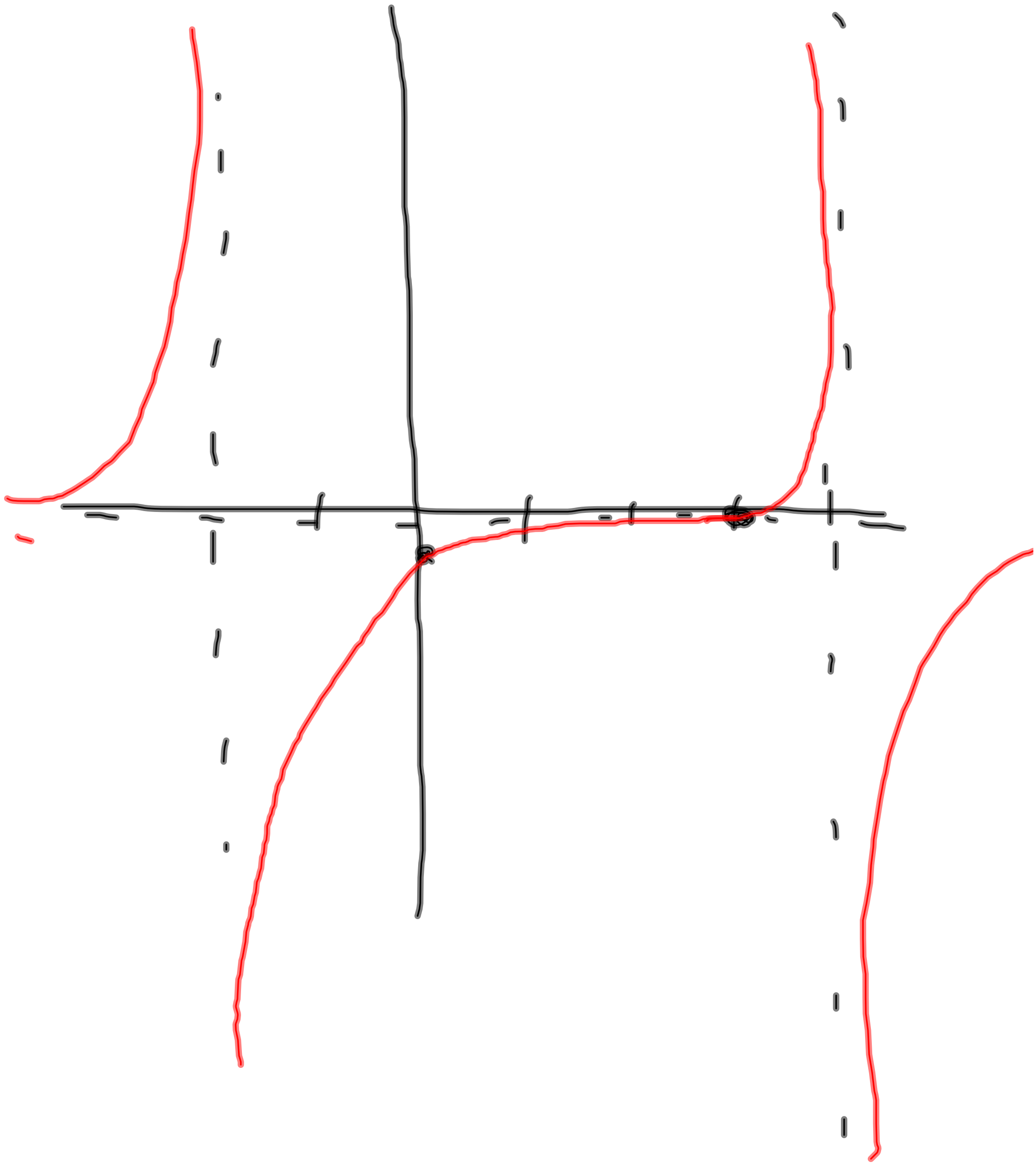
$\dfrac{+}{(-)(+)}$

+ + + $\quad$ − − − − $\quad$ − −

−10 $\quad$ −2 $\quad$ 0 $\quad$ 3 $\quad$ π $\quad$ 4 $\quad$ 10

$\dfrac{+}{(-)(-)}$ $\qquad$ $\dfrac{-}{++}$

$(-\infty,-2) \underbrace{\cup}_{\text{"union"}} (3,4)$



Graph $\dfrac{(x-1)(x+2)}{x(x-4)(x+3)}$

pos or neg?

→ cut points

$1, -2$ $0, 4, -3$

→ 1 pt in each interval

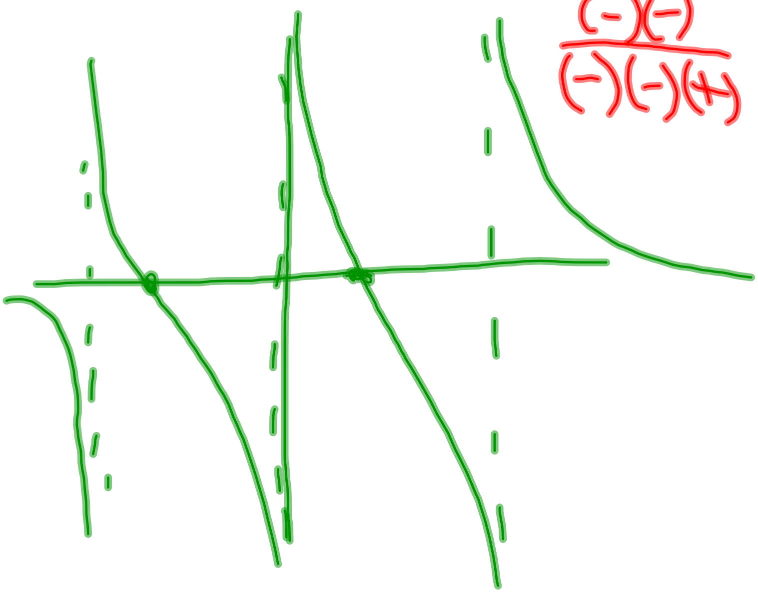
V A   Z   V A   Z   V A

$- - - \mid + + \mid - - \mid + + + \mid - - - \mid + + +$

$-10 \quad -3 \quad -2.5 \quad -2 \quad -1 \quad 0 \quad .5 \quad 1 \quad 3 \quad 4 \quad 10$

$\dfrac{(-)(-)}{(-)(-)(-)}$ $\dfrac{-+}{--+}$ $\dfrac{-+}{+-+}$ $\dfrac{++}{+-+}$

$\dfrac{(-)(-)}{(-)(-)(+)}$



HA

$y=0$

$\dfrac{x^2}{x^3} \Rightarrow \dfrac{1}{x}$